Analyse 1 - TD 1
NON CORRIGÉ

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Exercice 1

a)

$$\begin{aligned} & 5x + 2 \geq -3 \\ \iff \quad & 5x \geq -5 \\ \iff \quad & x \geq -1 \\ \iff \quad & x = [-1, +\infty[\end{aligned}$$

b)

$$\begin{aligned} & 2x - 1 < 4x + 3 \leq -x + 6 \\ \iff \quad & (2x - 1 < 4x + 3) \land (4x + 3 \leq -x + 6) \\ \iff \quad & (-4 < 2x) \land (5x \leq 3) \\ \iff \quad & (-2 < x) \land (x \leq \frac{3}{5}) \\ \iff \quad & x = \left]-2, \frac{3}{5}\right] \end{aligned}$$

c)

$$\begin{aligned} & |x - 1| < 4 \\ \iff \quad & (x - 1 < 4) \land (x - 1 > -4) \\ \iff \quad & (x < 5) \land (x > -3) \\ \iff \quad & x =]-3, 5[\end{aligned}$$

d)

$$\begin{aligned} & |x-2| \geq 3 \\ \iff \quad & (x-2 \geq 3) \vee (x-2 \leq -3) \\ \iff \quad & (x \geq 5) \vee (x \leq -1) \\ \iff \quad & x = \left]-\infty,-1\right] \ \cup \ \left[5,+\infty\right[\end{aligned}$$

e)

$$\begin{aligned} & |x-2| \leq |x| \\ \iff \quad & (x-2)^2 \leq x^2 \\ \iff \quad & x^2-4x+4 \leq x^2 \\ \iff \quad & 4 \leq 4x \\ \iff \quad & 1 \leq x \\ \iff \quad & x = \left[1,+\infty\right[\end{aligned}$$

f)

g)

$$\begin{aligned} & \sqrt{x+1} < 2 \\ \iff \quad & (x+1 < 4) \wedge (x+1 \geq 0) \\ \iff \quad & (x < 3) \wedge (x \geq -1) \\ \iff \quad & x = \left[-1,3\right[\end{aligned}$$

h)

$$\begin{aligned} & x^2+1 \leq 3 \\ \iff \quad & x^2 \leq 2 \\ \iff \quad & (x \leq \sqrt{2}) \wedge (x \geq -\sqrt{2}) \\ \iff \quad & x = \left[-\sqrt{2},\sqrt{2}\right] \end{aligned}$$

i)

$$\begin{aligned} & x^2+3x < 4 \\ \iff \quad & x^2+3x-4 < 0 \\ \iff \quad & (x-1)(x+4) < 0 \\ \iff \quad & (x < 1) \wedge (x > -4) \\ \iff \quad & x = \left]-4,1\right[\end{aligned}$$

j)

$$\begin{aligned} & x^3 - 3x^2 + 2x \geq 0 \\ \iff \quad & x(x^2 - 3x + 2) \geq 0 \\ \iff \quad & x(x-1)(x-2) \geq 0 \\ \iff \quad & \big((x \geq 0) \wedge (x \leq 1)\big) \vee (x \geq 2) \\ \iff \quad & x = [0, 1] \cup [2, +\infty[\end{aligned}$$

k)

Exercice 2

$$f : x \mapsto ax + b$$

$$\begin{aligned} & \big(|f(-1)| = 3\big) \wedge \big(|f(2) = 2|\big) \\ \iff \quad & (|b - a| = 3) \wedge (|2a + b| = 2) \\ \iff \quad & (b - a = 3 \vee b - a = -3) \wedge (2a + b = 2 \vee 2a + b = -2) \\ \iff \quad & (b - a = 3 \wedge 2a + b = 2) \\ & \vee (b - a = 3 \wedge 2a + b = -2) \\ & \vee (b - a = -3 \wedge 2a + b = 2) \\ & \vee (b - a = -3 \wedge 2a + b = -2) \end{aligned}$$

$$\begin{aligned} & b - a = 3 \wedge 2a + b = 2 \\ \iff \quad & 3a = -1 \wedge 3b = 8 \\ \iff \quad & a = -\frac{1}{3} \wedge b = \frac{8}{3} \end{aligned}$$

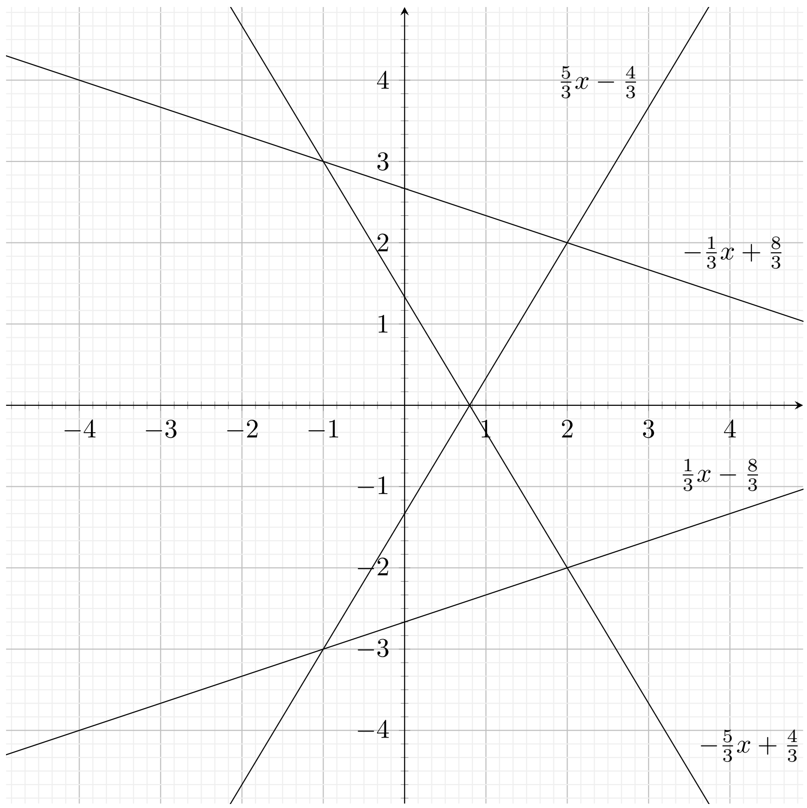
$$\begin{aligned} & b - a = 3 \wedge 2a + b = -2 \\ \iff \quad & 3a = -5 \wedge 3b = 4 \\ \iff \quad & a = -\frac{5}{3} \wedge b = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} & b - a = -3 \wedge 2a + b = 2 \\ \iff \quad & 3a = 5 \wedge 3b = -4 \\ \iff \quad & a = \frac{5}{3} \wedge b = -\frac{4}{3} \end{aligned}$$

$$\begin{aligned} & b - a = -3 \wedge 2a + b = -2 \\ \iff & 3a = 1 \wedge 3b = -8 \\ \iff & a = \frac{1}{3} \wedge b = -\frac{8}{3} \end{aligned}$$

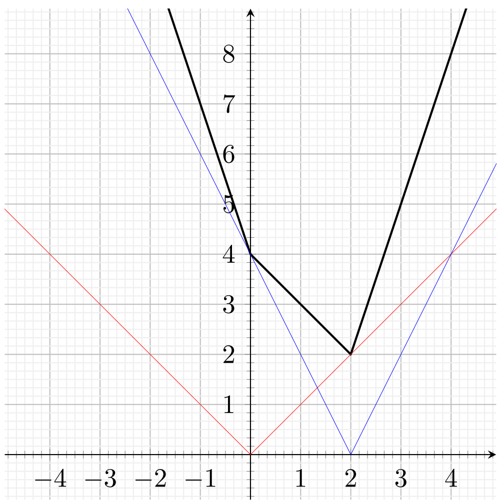
Donc

$$(a, b) \in \left\{ \left(-\frac{1}{3}, \frac{8}{3}\right), \left(-\frac{5}{3}, \frac{4}{3}\right), \left(\frac{5}{3}, -\frac{4}{3}\right), \left(\frac{1}{3}, -\frac{8}{3}\right) \right\}$$



Exercice 3

a)



b)

L'ensemble $f(\mathbb{R})$ est égal à $[2, +\infty[$.
La fonction f est minorée et elle n'est pas majorée.

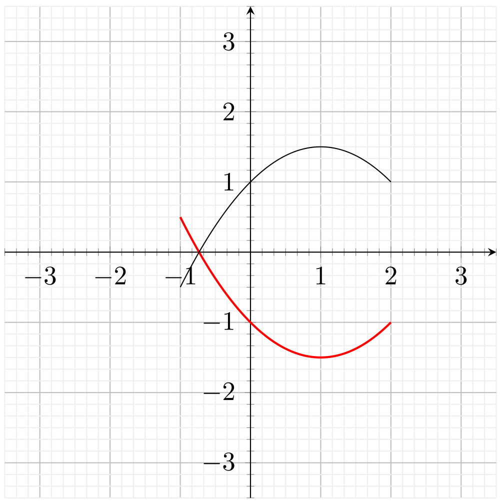
c)

Les antécédents par f de 3 sont 1 et $\frac{7}{3}$.
1 n'a pas d'antécédents par f.
L'antécédent par f de 2 est 2.

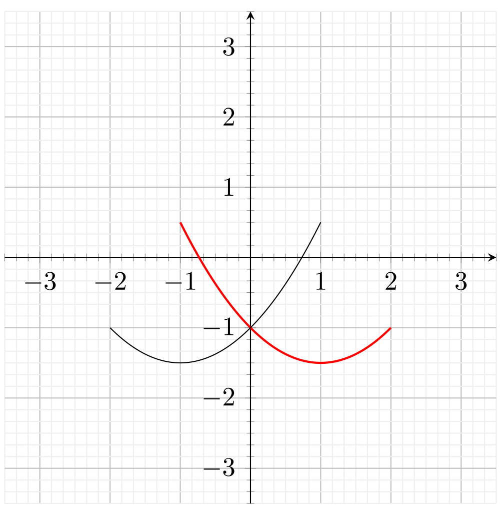
Exercice 4

a)

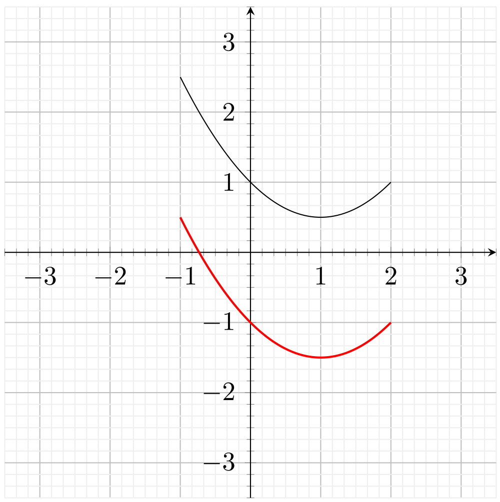
i)



ii)



iii)



iv)